647510 **MSC3MatC303x** **Seat No :________**

M.Sc. (Maths) Semester - 3 (*CBCS*) Examination

DEC/JAN. - 2020

Number Theory – 1 (Core)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1(a) If a is an odd integer then prove that $a^2 + (a+2)^2 + (a+4)^2 + 1$ is divisible by 12. **4**

Q.1(b) **Answer any two question out of three.** **10**

(1) Solve $158x - 57y = 7$

(2) Show that $[a, b].(a, b) = |a.b|$.

(3) Prove that number of primes of the form $4k + 1$ is infinite.

Q.2(a) Show that $11/5^{38} - 4$. **4**

Q.2(b) **Answer any two question out of three.** **10**

(1) If $ca \equiv cb\ (mod\ n)$, then show that $a \equiv b \left(mod\ \frac{n}{d}\right)$, where $d = \gcd(c, n)$.

(2) State Chinese remainder theorem and hence find the smallest solution which leaves remainders $2, 3, 2$ when divided by $3, 5, 7$ respectively.

(3) The linear congruence $ax \equiv b(mod\ n)$ has a solution if and only if ${}^{d}/_{b}$, where $d = \gcd(a, n)$.

Q.3(a) If the integer a has order k modulo n then show that $a^i \equiv a^j (mod\ n)$ if $i \equiv j\ (mod\ k)$. **4**

Q.3(b) **Answer any two question out of three.** **10**

(1) If n has a primitive root, then show that it has exactly $\phi(\phi(n))$ primitive roots.

(2) If the integer a has order k modulo n and $h > 0$, then a^h has order ${}^{k}/_{\gcd(h,k)}$ modulo n.

(3) If p is a prime number and ${}^{d}/_{p-1}$ then the congruence $x^d - 1 \equiv 0\ (mod\ p)$ has exactly d solutions.

Q.4(a) For $n > 1$, the sum of the positive integers less than n and relatively prime to n is $\frac{1}{2}n\phi(n)$. **4**

Q.4(b) **Answer any two question out of three.** **10**

(1) Determine the highest power of 5 dividing 1000! and the highest power of 7 dividing 2000!.

(2) In usual notation, show that $\prod_{d/n} d = n^{\frac{\tau(n)}{2}}$.

(3) State and prove M$\ddot{o}$bious inversion formula.

Q.5(a) If p_n is the n^{th} prime number, then show that $p_n \leq 2^{2^{n-1}}$. **4**

Q.5(b) **Answer any two question out of three.** **10**

(1) For any integer $n \geq 1$, prove that $\sum_{d/n} \mu(d) = \begin{cases} 1, & \text{if } n = 1 \\ 0, & \text{if } n > 1 \end{cases}$.

(2) Define greatest integer function. Show that $[x] + [-x] = \begin{cases} 0, & x \in Z \\ -1, & x \notin Z \end{cases}$.

(3) If $\gcd(a, b) = 1$then show that $\gcd(2a + b, a + 2b) = 1\ or\ 3$.

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